ever seen at any voltage. That breakdown within the electrode system actually occurred was evident in many cases by direct visual observation, although the precise location of a given discharge could seldom be determined.

From the data given herein it seems reasonable to conclude that the initiating mechanism of high-voltage electrical breakdown *in vacuo* is not a process whereby charged particles reproduce themselves by a series of secondary chain reactions involving electrons striking the anode, with an efficiency which depends continuously on voltage and which rises to unity at breakdown. Note that no direct statement can be made about chain reactions not involving electrons, as, for instance, a positive ion-negative ion reaction, for which in one case over-all efficiencies as high as one percent have been observed.[7] Aside from the latter type of reaction, the mechanisms left as possibilities appear to be those triggered by random events originating within or without the gap, such as electrostatically detached clumps,[5] excitation from some external source, e.g., cosmic rays, or cumulative resistive heating at field-emitting points of high-current density.[2]

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# Temperature-Dependent Equations of State of Solids

J. J. GILVARRY

*Research Laboratories, Allis-Chalmers Manufacturing Company, Milwaukee, Wisconsin*

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An isothermal equation of state of a solid is considered, which contains as special cases the equations of Birch, Murnaghan, Bardeen, and others. The equation is generalized to arbitrary temperature by replacing two constants of the equation by temperature-dependent parameters, whose functional form is determined by considerations of thermodynamic consistency. The thermal properties of the solid implied by this equation of state are examined. It is shown that the generalized equation is consistent with the Mie-Grüneisen relation for the thermal pressure of the lattice, and that the corresponding Grüneisen parameter is only slightly dependent on temperature, in general. The form of the generalized equation of state at low temperature is exhibited as an explicit function of volume and temperature for a solid whose heat capacity obeys the Debye law. A comparison with pressure-volume data of Swenson for potassium at low temperatures shows excellent agreement of the generalized equation of state with experiment.

## I. INTRODUCTION

SINCE the time of Murnaghan's successful application of the theory of finite strain to obtain equations of state for solids at high pressure,[1-3] considerable attention has been devoted to the general problem he initiated. Additional equations of state have been derived by later investigators from Murnaghan's formulation of the theory of finite strain.[4-7] The salient example is perhaps the equation of Birch,[5] which has been used very successfully to represent pressure-volume relations for solids, at large pressures[8] and at constant temperatures extending to low values.[9] The relation has been used widely in geophysical applications, for extrapolation of compression data for elements and compounds beyond the experimental range, and in the inference of physical properties of the earth's interior.[8,10,11] Murnaghan's equation has been applied in a discussion of the effect of shock waves on solids.[12]

A wider development has proceeded in the derivation of equations of state of solids from quantum mechanics and from lattice models. As an example among many in the former class, one can cite Bardeen's equation.[13] In principle, the limitations on accuracy in the quantum-mechanical calculation of an equation of state are set only by mathematical complication in the solution of

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